3D Maxwell Equations

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I give in my docdb notes the 3D equations for the "2D multipoles" [1]. The 3D "dipole" which satisfies Maxwell's equations is:

$$\frac{B_y}{B_0} = 1 - \sum_{N=1}^{\infty} k_N \sin\left(\frac{Nz}{R}\right) \sinh\left(\frac{Ny}{R}\right) + \cdots \qquad (1a)$$

$$\frac{B_z}{B_0} = \sum_{N=1}^{\infty} k_N \cos\left(\frac{Nz}{R}\right) \cosh\left(\frac{Ny}{R}\right) + \cdots \qquad B_x = 0 \qquad (1b)$$

z is the azimuthal direction, y is vertical, and x is radial. The $\cdots$ denotes the other trig-hyperbolic-trig combinations, which aren't important for this study.

The Taylor series expansion is:

$$\cosh\left(\frac{Ny}{R}\right) = 1 + \frac{N^2y^2}{2R^2} + \cdots \qquad \sinh\left(\frac{Ny}{R}\right) = \frac{Ny}{R} + \cdots \qquad (2)$$

Whether it is $\cosh\left(\frac{Ny}{R}\right)$ or $\sinh\left(\frac{Ny}{R}\right)$ depends on the shape distortion of the pole pieces that we have – see Appendix I. For E821, we had predominantly equ. 1a-b – see Appendix II.

We didn't measure the longitudinal field for E821, but we did for E989 – see Fig. 1. It looks like the leading term is the $N = 1$ term: $\approx 50ppm\ sin(z/R)$. Note that equ. 1a and 1b have the same k_N.

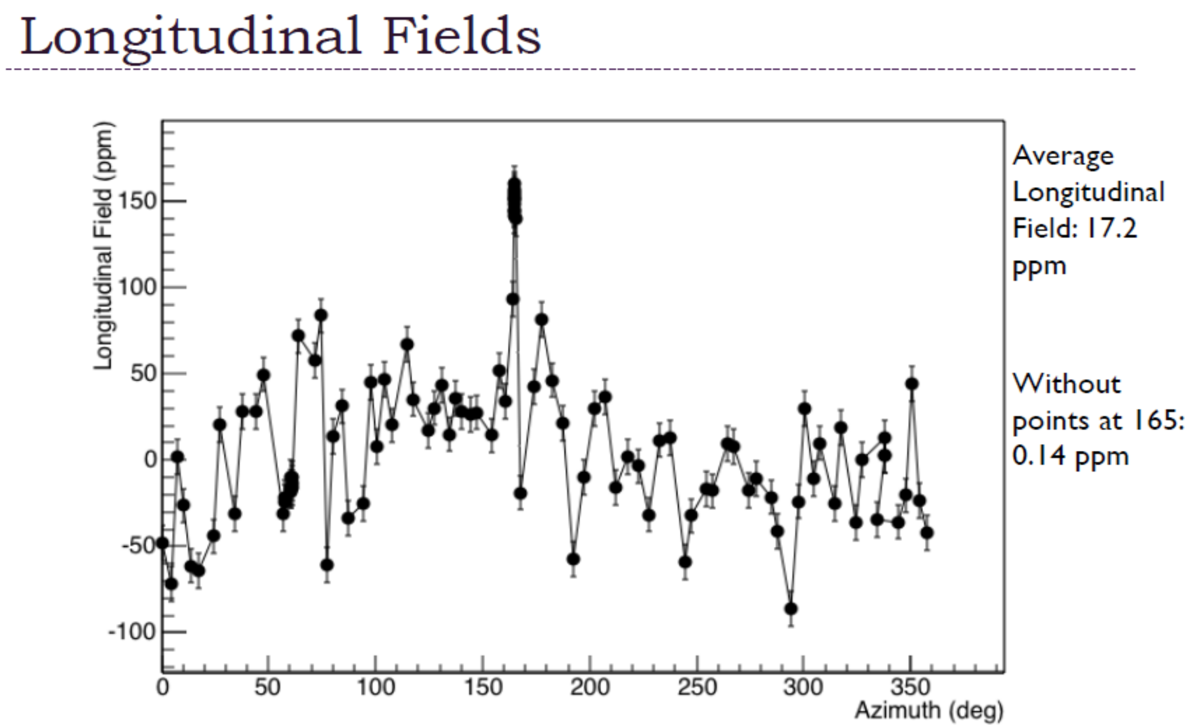

Fig. 1. Ref. 2 longitudinal magnetic field around the ring. Looks like $\approx 50ppm\ \sin(z/R)$. Note that equ. 1a and 1b have the same k_N.

References

1. W. Morse, Docdb 2447, 2454, 18185.
2. R. Osofsky et al., Docdb 4671.

Appendix I

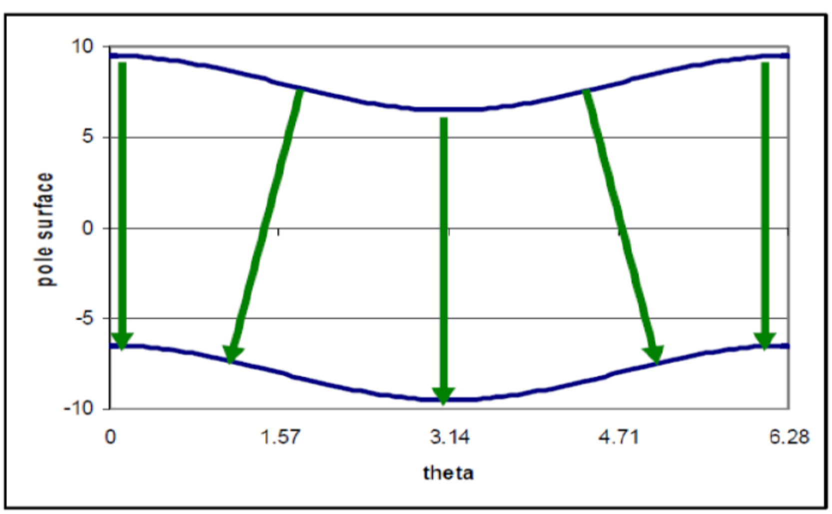

Fig. 1A.Vertical magnetic field component that goes as $\sinh\left(\frac{Ny}{R}\right)$. $\theta = z/R$.

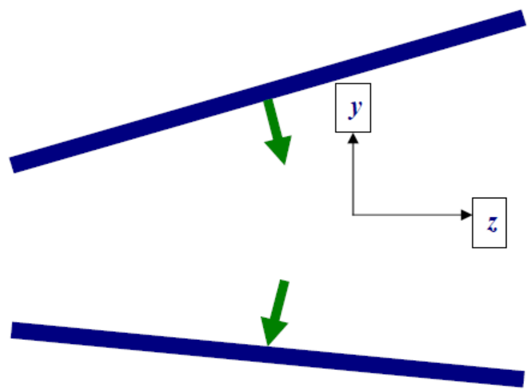

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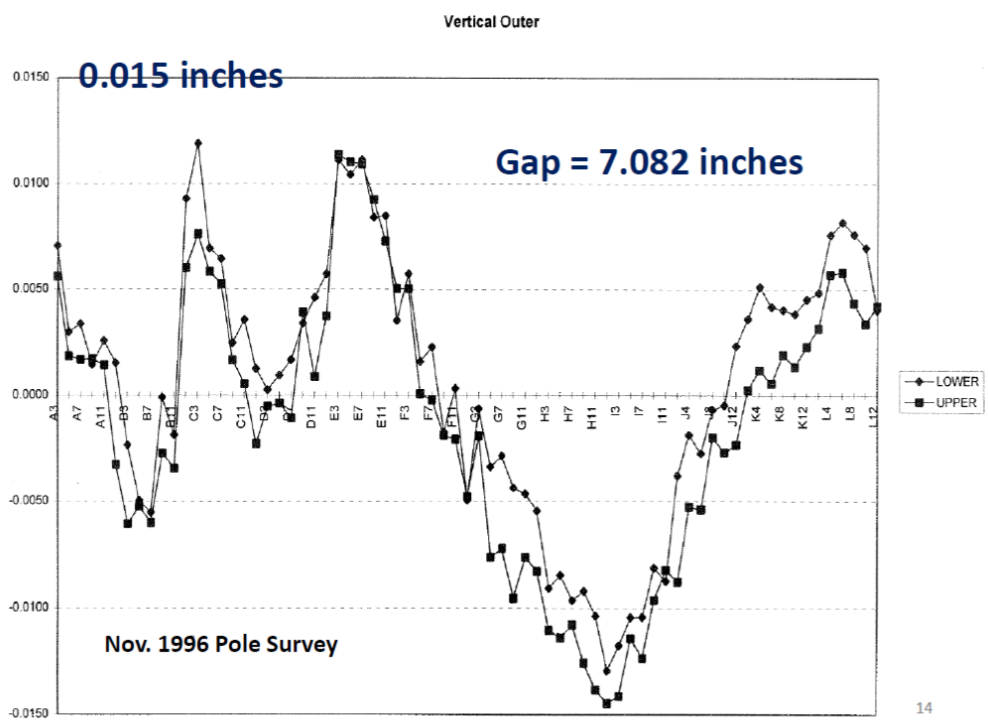
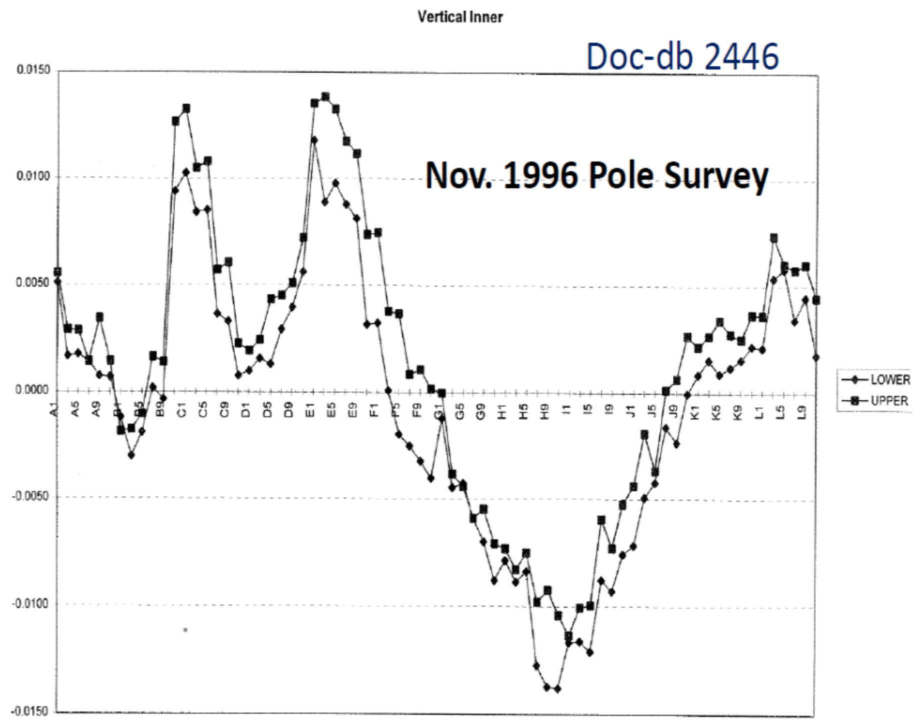
Fig. 2A. Vertical magnetic field component goes as $\cosh\left(\frac{Ny}{R}\right)$.

Appendix II – E821 Surveys





Survey Fourier Analysis