Optimizing muon injection for the Muon g-2 experiment

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Abstract:

The Muon g-2 experiment seeks to measure the anomalous magnetic moment (q-2)of muons to a .14 ppm uncertainty. In order to accomplish this, the experiment must be supplied with the largest number of muons with energies within .2% of 3.094 GeV/c possible. This work used the Geant4beamline tracking program to simulate and examine various beam scenarios, including the use of a wedge, wedge offset, beam offset at injection, inflector design, and initial beam energy in order to optimize the final number of measurable muons and therefore reduce the systematic uncertainty of the measurement. It was discovered that a beam offset at injection of -67.1mm for an unobstructed beam and -65.1mm for a wedge-in (0mm) beam optimized muon transmission through the inflectors. It was also found that inserting a wedge 3mm into the beam path while raising the initial energy by .8% can ultimately increase the experimental yield by about 4%. However, many wedge offset and energy increase combinations produce comparable values to this, and thus it was concluded that while wedge use can improve experimental yield, most improvement will result from the use of an inflector without material on its longitudinal ends; which was shown to increase yield by over 50% as compared to the current inflector used by Fermi National Accelerator Laboratory.

1. INTRODUCTION

1.1. Background

This work was conducted at the Fermi National Accelerator Laboratory with the goal of providing insight about various aspects of the muon beam used in experiments in order to maximize experimental yield. Before the experimental process is explained, it is necessary to give background as to the experiment to which this work pertains.

This work pertains primarily to the Muon g-2 experiment. The Muon g-2 experiment is an ongoing, international collaborative project based at the Fermi National Accelerator Laboratory (FNAL), in Batavia, IL, USA. It seeks to measure the anomalous magnetic moment of muons. The magnetic moment of an object can be defined to be a measure of the strength of said object's magnetism. The spin magnetic moment is a combination of various particle-specific parameters, which can be observed in the equation below: [1]

$$\mu = g(\frac{q}{2m})S$$

where μ is defined to be the spin magnetic moment of a Dirac particle, g the gyromagnetic ratio (or g-factor, which will be mentioned later in this introduction), q is defined to be the charge of a particle, m is defined to be the mass of the particle, and S the value of the spin angular momentum.

The Dirac equation predicts the spin g-factor (hereafter referred to as g) for electrons and other leptons to be 2, and quantum electrodynamics predicts it to be slightly larger than 2. This difference is referred to as the anomalous magnetic moment of particles, and is represented by

$$a = \frac{g-2}{2},$$

where a is the anomalous magnetic moment, and g is the measured value from experiment or that predicted from quantum electrodynamics. Previous experiments have measured the *electron's* spin g to be 2.00231930436256(35), with an accuracy of .76 parts per trillion (ppt) [2]. Its difference from 2, or anomalous magnetic moment (a), is explained by quantum electrodynamic theory. However, the previous measurement of the muon magnetic moment at Brookhaven National Laboratory (BNL) was only calculated to an accuracy of .7 parts per million (ppm) and an uncertainty of .54 ppm. What's more, the muon's a is not completely explained by quantum

electrodynamic theory. The BNL measurement differed from theoretical predictions with a certainty of over 3σ -a difference not sufficient to be classified as a discovery, but enough to necessitate further experimentation [3]. The muon's anomalous magnetic moment needs to be more accurately measured in order to determine whether or not the anomaly can be explained through the current quantum mechanical framework. The Muon g-2 project at FNAL seeks to measure the anomalous magnetic moment of the muon to an uncertainty of .14 ppm, or 5σ , as confirming the results of the BNL experiment to such a degree of certainty would point to the existence and effects of new physics beyond the Standard Model affecting the muons measured.

In order for the Muon g-2 project to measure the muon's g to said accuracy and certainty, the experiment must be supplied with a very large number of measurable muons. The work detailed in this paper seeks to improve upon just that. The Muon g-2 experiment operates on the following layout:



Figure 1: A diagram of the layout of the Muon Campus at Fermilab. [4]

Muons are first produced via proton collisions at the AP-0 Target hall. The products from these collisions are then passed through the Delivery Ring (DR). After four revolutions around the DR, the secondary protons are removed from the beam by a kicker magnet. The now-purified muon beam is then passed through the M4/M5 line, and then, for the Muon g-2 experiment, through the M5 line and into the MC 1 Experimental Hall, where the muon storage ring (SR) is housed. A superconducting magnet, known as the inflector, temporarily shields the 1.45 T magnetic field of the SR in order to allow the muon beam to enter. Once in the SR, the muons are subjected to said vertical magnetic field that allows them to be circulated through the ring with minimal loss. During measurement, the muons make a designated number of turns around the storage ring, after which data will be collected.

1.2. The motivation of this work

Given that the statistical uncertainty of the SR measurement is $1/\sqrt{N}$, where N is the number of muons inside the SR, it is necessary to supply the SR with as many measurable muons as possible to increase accuracy and certainty in the measurement. There exists a few challenges to this with the current experimental apparatus. Firstly, the storage ring only accepts muons within approximately .2% of 3.094 GeV/c–a value referred to as the 'magic momentum' (MM). However, the DR produces a beam of muons with a 1% rms momentum spread [5]. Thus, approximately 90% of the beam is lost through collimation in the SR, and is therefore immeasurable.

A Boron Carbide wedge was installed directly after the DR in the beam path in order to mitigate the problem of the momentum spread, and is represented in Fig. 2 below:



Figure 2: A representation of the wedge in the beamline between a dipole (left) and a quadrupole (right), from a G4beamline simulation. The wedge is approx. 80mm long from the point to base (the vertical direction in this representation), and approx. 130mm from side to side (the horizontal direction in this representation). The wedge absorber has a full angle of 146°. [5]

The wedge offset-or degree to which the wedge penetrates the beam-is a parameter that can be manipulated, with 0mm offset being defined as the wedge covering half of the beam. A positive offset refers to further insertion of the wedge into the beam path, and a negative offset therefore refers to an insertion that covers less than half of the beam. The wedge and beam are oriented in a way such that muons with energies around or lower than 3.094 GeV/c will not pass through the wedge at all, and muons with energies higher than 3.094 GeV/c will pass through the wedge to a degree proportional to their distance from that value. Thus, the wedge is able to decrease the rms momentum spread by slowing the muons with higher-than-MM energies. However, as can be seen in Fig. 3, the wedge also decreases the overall average momentum away from the MM, which can be observed in the drop in the average muon momentum from 3088 MeV/c to 3075 MeV/c.



Figure 3: Left: A histogram of the momentum spreads within 3% of the MM at the end of the M4/M5 line for the wedge-in (0mm offset) and wedge-out beams. 'Cent' refers to the peak of each histogram, which represents the momentum bin containing the highest number of particles–it is loosely analogous to the 'mode'. Right: A visual representation of the concept of wedge offset. [5]

Thus, while the wedge mitigated the problem of the large momentum spread, it reintroduces the problem of a low average momentum, and therefore the challenge of supplying the SR with as many measurable muons as possible. Measurements of the effects of various wedge offsets on the number of muons inside the storage ring needed to be taken in order to balance the factors of creating a smaller momentum spread while maintaining a higher average momentum. A discussion of this will be occur later in this paper.

As mentioned previously, the inflector (pictured below) momentarily shields the magnetic field of the SR in order to allow the muons to be injected into the apparatus.



Figure 4: Left: A drawing of a cross section of the SR, showing the inflector [6]. Right: A sketch of the inflector only that demonstrates the use of ionizing material on its longitudinal ends. The inflector is approx. 1.7m long.

However, the current inflector is constructed with physical material obstructing the beam path on both the beam line-facing end, as well as the injection point end. This causes additional muon loss due to scattering. The Muon g-2 project understands the issues with the current inflector and wishes to install a new inflector—without material on either ends—in order to reduce scattering and therefore increase the number of muons injected into the SR. However, prior to this work, it was not known the degree to which the new inflector would improve experimental yield (if at all). This paper will also detail a comparison study of the old and new inflectors in subsequent sections.

Thirdly, there also exists a parameter known as the beam offset that required optimization. If the beam and beam line were said to be traveling in the z-direction, taking a cross section would reveal that the beam is not necessarily traveling along the z = 0 axis with respect to the beam pipe. This is what is called the beam offset-the distance between the beam and the center of the beam pipe. The beam offset is a parameter that can be controlled-and therefore, optimized. This work will also study the interplay of the wedge and beam offset in order to determine the optimal offset by which the beam center aligns with the inflector's center. This is the point of maximum transmission and therefore allows the highest number of MM muons to enter the SR.

In short, the goal of this work is to minimize the statistical uncertainty of the measurement of the anomalous magnetic moment of muons by studying and optimizing the in-tandem effects of three variable parameters: the wedge offset, the beam offset, and the old and new inflectors.

2. METHODS

This work was conducted using the Geant4beamline v. 3.06 program in the National Energy Research Scientific Computing Center (NERSC). In order to itemize our approach, the M4/M5 line, new inflector, old inflector, and SR were all simulated with different tracking codes and were run separately. The beam would first be tracked from the exit of the DR, through the wedge, and until the end of the M4/M5 line. The output from this simulation would then be passed through an inflector simulation, allowing the continued tracking of the same beam. Next, the output from that same inflector simulation would be tracked through the SR, the output of which would be used to draw conclusions and assess the efficacy of the beam parameters in question, with efficacy being assessed on the number of muons surviving to 50 turns around the SR.

3. DATA ANALYSIS AND DISCUSSION

3.1. Initial Wedge Offset Assessments

We will begin the discussion with the topic of the wedge, as it is both first in the beam line, and the chronologically first to be studied in this work.

Initial wedge simulations were taken with the default G4beamline beam offset, -63.1mm, as well as at 0% increase in initial beam energy. All simulations were conducted with 400,000 particles in the beam. Wedge offsets ranged from -4mm to +5mm, in 1mm increments. In these initial assessments, each wedge offset was not tracked through the SR. Rather, these preliminary assessments of wedge efficacy were taken from the distributions produced at the end of the M4/M5 line. In each plot, cuts were made to only include muons with energies within $\pm 3\%$ of the MM in order to eliminate the effects of outliers.



Figure 5: Left: Two plots: the average total momentum as a function of wedge offset measured on the left y-axis and in fuchsia; and the variance in the beam momentum measured on the right y-axis and in green. N/W' refers to the wedge-out case. The total momentum was used to calculate the variance. Note that in this representation, the 5mm and N/W points for each plot coincidentally fall in the same location on this graph. This occurred purely by chance and is not necessary, as the two plots are measured on different scales. Right: Two plots showing the percent increase in the number of MM muons (orange parabola, left axis) as a function of wedge offset, as well as the decline in energy (blue linear, right axis) that results from greater wedge insertion. When the momentum decrease becomes too great and begins to 'overpower' the benefit provided by the narrowed peak, the efficacy of the wedge begins to decline. The wedge accomplishes peak performance when placed at a 1mm offset.

Under these circumstances, it was found that the wedge did succeed in reducing the spread in muon momentum, which can be observed in the distinct downward trend in the beam momentum variance in Fig. 5 and was the desired outcome. However, as can be seen in Fig. 5 on the following page, the wedge also decreases the average total (x, y, and z; 3-dimensional) momentum, which is undesirable. A decrease in the average momentum is undesirable because it slows muons away from the MM value, introducing a greater potential for a greater number of muons to have energies outside of the range of SR tolerance and therefore, not allowed to be measured.

The results represented in Fig. 5 are what was expected from this work, as these results agree with those of past initial studies on wedge function and efficacy [5][7]. It is clear that as of the end of the M4/M5 line, wedge use succeeded in reducing the momentum spread. This is encouraging, as the original intention for the wedge was to raise the number of measurable muons in the SR by narrowing the momentum spread of the beam and therefore reducing the approx. 90% muon loss that resulted from the large spread. On that front, our first results point towards high wedge efficacy. However, as it stands now, the same physics that reduce the momentum spread also decrease the average momentum. Wedge geometry and placement forced the more energetic section of the beam through material, so wedge use necessarily decreased the average momentum by cutting out the leading tail of the wedge-out curve in Fig. 3, which contained higher-than-MM muons. Thus, the overall average total momentum of the wedge-in beam is decreased along with the variance in the total momentum. The decrease in the average total momentum is undesirable, as it greatly reduces the number of muons within .2% of the MM delivered to the SR and therefore, reduces the number of muons that will be accepted by the SR and measured. This is counterproductive to the wedge's intended purpose of raising the number of MM muons. This conundrum provided the motivation for what will be the final topic of this work: a study in increasing the initial beam energy. These results spurred the idea that raising initial beam energy would give us the 'best of both worlds' of the two plots above: both a reduced momentum variance and a beam of a momentum close to the MM. This work performed preliminary investigations of this claim, which will be discussed later in this section when we will discuss SR assessments.

It should be noted that systematic error bars are too small to be viewed on these plots, as the systematic error in the momentum measurements only gives $\pm 1.58 * 10^{-3}$ MeV/c. Given that the measurements are taken on the order of magnitude of 10^3 MeV/c, the statistical uncertainty is 6 orders of magnitude smaller than the measurements, rendering them impossible to be seen on this and most other plots in this work.

3.2. Optimization of injection

We now turn our discussion to the beam offset and, necessarily, the inflectors. When entering a particle distribution as input for an inflector simulation, the parameter 'beam offset' can be specified. As mentioned previously, -63.1mm was initially set as the default. Prior to this work, it was not known which beam offset would give the highest number of muons within .2% of the MM, or if the beam offset at injection would have an effect on said output. Thus, before conducting a thorough comparison study of the old versus new inflectors, it was necessary to take some preliminary measurements regarding the effect of beam offset on MM muon yield through both inflectors, in order for them to be compared in their optimized state.

This analysis was intended to be rough and preliminary, and thus four classes of simulations were taken: various beam offsets for a wedge-in (0mm offset) beam through the new inflector, various beam offsets for a wedge-out beam through the new inflector, various beam offsets for a wedge-in beam through the old inflector, and various beam offsets for a wedge-out beam through the old inflector (the scenario originally present in the experiment). Results were taken for the number of muons within 1% of the MM, due also to the fact that a slightly broader range is allowed by the inflector than the SR. Simulations of the wedge-out case were run for beam offsets ranging from -59.1mm to -71.1mm, in increments of 2mm. This range was a result of the desire to continuously take measurements at $\pm 2mm$ increments from -63.1 until a definite peak in the number of MM muons could be observed. Simulations for the wedge-in case were taken in a shorter range because a suitable and predictable peak could be extrapolated from that produced by simulations ranging from -69.1mm to -63.1mm. Each measurement was taken at the injection point of the inflector and SR. A plot of the results can be seen in Fig. 6 below:



Figure 6: A scatter plot of the number of muons within 1% of the MM after each run through the inflectors for different beam offsets.

It can be clearly observed that -67.1mm offset was the optimal value for both wedge-out cases, and -65.1mm offset that for both wedge-in cases. These two values were used in all future simulations in order to compare the four beams in their optimal states. However, it is important to note that the largest number of MM muons from the previous four beam scenarios results from the new inflector, wedge-out beam scenario; with the new inflector, wedge-in beam close behind at it's highest count. This is important because again, the purpose of the wedge is *raise* the number of MM muons, and yet here it appears to lower that value by about 2%. The statistical uncertainty for the values represented in Fig. 6 is at most approx. $\pm 3E - 3$, which is less than 1 and therefore not very meaningful to data consisting purely of counts. The muon counts for the wedge-in, new inflector and wedge-out, new inflector are statistically different; but are at the same time very comparable qualitatively. This provides additional motivations for studies in raised beam energy. It follows from our understanding of the interaction between the wedge and the beam that the number of MM muons for the wedge-in case should be lower, as the wedge has lowered the average energy of the beam. However, even with this, the wedge-in beam with optimized offset and through the new inflector. Could raising the beam energy drive the MM muon count to

a value higher than that of the wedge-out beam, therefore allowing us to both reduce loss while increasing the number of MM muons? At this point, we were very curious to investigate whether raising the initial energy of the wedge-in beam would increase the number of MM-muons beyond the wedge-out, new inflector value.

Nevertheless, although we were simply studying beam offsets in order to *later* assess inflector efficacy, an improvement due purely to the use of the new inflector is already evident. In both wedge-in and wedge-out beams, the new inflector yields decidedly more MM muons than the old inflector case, with the optimized wedge-in, new inflector scenario giving approx. 17% more MM muons than the optimized old inflector case–a very large and exciting improvement. The new inflector, wedge-in case also yields approx. 10% more MM muons than the old inflector. Again, given the almost meaninglessness of statistical uncertainty in these simulations, these are exiting preliminary results pointing to high efficacy of the new inflector by itself.

3.3. A Study of the Two Inflectors

Now that we can compare both inflectors in their optimized state, we can conduct a far more valid comparison study of the old versus the new inflector.

The inflector is a necessary component of the experimental apparatus. Were it not for the inflector, incoming muons would simply be deflected by the 1.45 T magnetic field of the SR and would therefore never be allowed to enter the SR. The inflector shields the muons inside of it from the magnetic field in order to facilitate their injection. As mentioned previously, the difference between the old and new inflectors is that the new inflector lacks the physical material on the longitudinal ends that is present in the old inflector (a schematic of the inflector inside the SR can be seen in Fig. 4). The old inflector causes loss *in addition* to the problem of muon energy spread due to the fact that muons passing through the material ionize said material and therefore lose energy from the MM value [8]. This is the same process by which the energy spread is decreased by the wedge, but instead is occurring in a location and manner that is problematic to the experiment, as this energy loss can cause otherwise-MM muons to be rejected by the SR. Thus, the Muon g-2 collaboration wishes to install a new inflector, that will not include material on the longitudinal ends of the inflector. Prior to this work, the degree of improvement of the new inflector was only approximately known. Thus, it was necessary to conduct a comprehensive comparison study of the two.

In this subsection, we will be discussing simulations that only track particles from the end of the M4/M5 line to the injection point of the SR. Plots of the momentum spreads for the four beams (combinations of old inflector, new inflector, wedge-in, and wedge-out) can be seen below.



Figure 7: Left: a plot comparing the momentum spread of the old inflector (cyan) vs. that of the new inflector (fuchsia) for the wedge-in beam. Right: a plot comparing the momentum spread of the old inflector (orange) vs. the new inflector (blue) for the wedge-out case. Both conducted with 400,000 particles. Data taken from the injection point, immediately downstream of the inflector. The bump centered around 2300 MeV/c is due to the fact that aperture effects were included in these simulations.

The momentum distribution of the new inflector beams in both cases is visually shifted right, towards higher energies. This can also be observed quantitatively by upward shifts in both the average momentum and the central momentum. The average momentum shifts upwards by approximately 5-6 MeV/c in each graph, and

the central momentum shifts upwards by a comparable value as well. This is a promising result as to the efficacy of the new inflector. The average energy increases for both the wedge-in and wedge-out cases are of comparable values as well, but the energy of the wedge-in beams remains lower (approx. 3073 MeV/c at most) than that of the wedge-out beams (approx. 3088 MeV/c at most). Again, this provided additional motivation for the studies in increased beam energy that will be discussed in the next section. Would increasing the beam energy, in combination with the new installation of the new inflector and wedge, push the momentum spread farther towards higher energies, and therefore, the MM?

In Fig. 8 is a graph of the number of muons within various percentages of the MM for the wedge-in and wedge-out beams. The new inflector, wedge-out case increases the number of MM muons by approx. 6%. While this may not strike the reader as a large gain, at this point it is still a meaningful one. However, the percent increase in muons for the wedge-in beams is double that. It is extremely interesting that the number of MM muons for the old inflector, wedge-in case is higher than that of the old inflector, wedge-out case. It appears that while the wedge-in beam experiences double the improvement with new inflector use, it also gives said improvement from a higher baseline. This is again curious because we might have expected a lower MM muon yield from the wedge-in beam at this point, given the results from the previous subsection. Although this subsection does not primarily concern the wedge, it is especially important to note the previous two sentences, as those observations indicate the need for additional research. However, remember that these plots say nothing yet of the effect of the new inflector and/or wedge inside the SR. But nevertheless, the takeaway from this data was that there is great promise for both the wedge and the new inflector. In all cases, the new inflector improves MM muon-output buy statistically significant amounts–and even more so in the wedge-in case. This was an exciting result that points to the high efficacy of the new inflector.



Figure 8: Left: A plot of the gain in muons within various percents of the MM for the wedge-in (0mm offset) beam. Right: the same for the wedge-out beam. Both from data taken at the injection point of the SR.

Thus, at this point it is clear that the new inflector can be strongly effective at increasing the number of MM muons–which is, after all, the goal of this work.

3.4. All together now: wedge offset, beam offset, inflector, and energy considerations inside the SR

Now that we better understand how to optimize the beams to judge them fairly, as well as the effects of the individual beam components and variables, we may now address the question that has been present and reoccurring from the start: will raising initial beam energy increase wedge efficacy and overall experimental yield?

To answer this, the particle distributions produced at the end of the M4/M5line were run through a python script that normalized the x and y positions of each particle, as well as raised the energy of the beam by various percents. These raised-energy distributions were then tracked through an inflector simulation, and then directly piped through to be tracked 50 turns around the SR. Then, the distributions from this SR simulation would be analyzed for the number of MM muons. A plot of the results can be seen in Fig. 9 below.



Figure 9: A plot of the number of muons with momentums within .2% of the MM after 50 turns inside the storage ring, with all previous beam parameters considered.

These are the 'final' results of the cumulative effects of all the beam parameters that have been mentioned in this paper thus far. These are the final counts of MM muons inside the SR after 50 turns—the muons to which the experiment will actually have access.

The fuschia bar on the farthest left represents the beginning state of the Muon g-2 apparatus: the wedgeout beam through the old inflector, +0% energy. The black bar to the right represents the wedge-out beam through the *new* inflector, also +0% energy. The most noticeable improvement in the figure is that between the wedge-out beam through the old and new inflectors, with the new inflector ultimately yielding approx. 53% (over 5000 count) more MM muons than the old inflector. This is a large, exciting improvement that (taken with the previous data in this work) strongly points to the efficacy of the new inflector.

The grey bar represents the case of a wedge out beam through the old inflector, but with a .2% energy increase. This simulation was performed due to the fact that the material on either ends of the inflector slows the muons by about 6 MeV/c (which can be seen in Fig.7). We wanted to test whether or not the aforementioned 50% increase in MM muons in the SR was due solely to the fact that the old inflector slows said muons. Thus, we ran a simulation in which the beam energy was raised by .2% (which corresponds to approx. 6 MeV/c) in order to compensate for this energy loss and truly assess whether or not the improvement was due to the new inflector only produced approx. 500 more muons than the +0% beam through the old inflector, and so it was comfortably concluded that the new inflector does provide a significant increase the number of MM muons, even with energy loss considerations.

As for the wedge alone, the beam functions as expected in the +0% energy simulations—the number of MM muons decreases with wedge insertion, as greater wedge insertion should lower the beam energy due to a greater 'portion' of the beam ionizing the wedge [7].

We then turn to the section of Fig. 9 representing the wedge-in beams of increased energies. While it is noticeable and statistically significant that each energy increase for each wedge offset gives a larger number of MM muons, the increases are unfortunately only the size of 100-200 muons (less, for the +.6% case). This is much smaller than we had hoped. The largest number of MM muons results from the +.8%, +3mm offset case, at 16,062 muons, which is a 3.75\% increase from the wedge-out, +0%, new inflector scenario. This is still an increase, and a statistically meaningful increase at that. However, it is not as large an increase as was originally speculated, especially given that wedge use gave high increases in MM muons at previous points in the beam line. Unfortunately, it does not appear that those large increases necessarily carry over to 50 turns inside the storage ring.

4. SUMMARY

While previous assessments of the wedge and inflectors have been made, none have been carried through to the end of the experiment. The initial data taken before the SR indicated that the wedge has the potential to provide double-digit increases in experimental yield of up to about 27%, and that the inflector would provide more modest improvements, of a maximum of 17%. However, tracking the distributions used to make those conclusions to the end of the experiment reveals a different outcome: that the new inflector without the wedge can improve experimental yield by about 52%, and that the wedge would only add about 3.8% more MM muons in addition to that. While the results are not as dramatic as was initially hoped for the wedge, it was still concluded that it does have an effect on the experimental outcome, and raising the energy does in fact raise the yield–albeit, not as drastically as initially predicted. However, the new inflector–when optimized–has been shown to surpass expectation in its ability to increase experimental yield and therefore, accuracy (even with energy loss considerations). Nevertheless, we have found that it is possible to use tools already at our disposal (namely, the wedge, modest energy increases, and the new inflector) to greatly increase certainty in the measurement of the muon's anomalous magnetic moment.

References

- [1] C. S. Bogdan Povh, Klaus Rith, F. Zetsche, Particles and Nuclei: An Introduction to the Physical Concepts, Springer Science Business Media.
- [2] B. Odom, D. Hanneke, B. D'Urso, G. Gabrielse, New measurement of the electron magnetic moment using a one-electron quantum cyclotron, Phys. Rev. Lett. 97 (2006) 030801. doi:10.1103/PhysRevLett.97.030801. URL https://link.aps.org/doi/10.1103/PhysRevLett.97.030801
- [3] G. W. Bennett, B. Bousquet, H. N. Brown, G. Bunce, R. M. Carey, P. Cushman, G. T. Danby, P. T. Debevec, M. Deile, H. Deng, S. K. Dhawan, V. P. Druzhinin, L. Duong, F. J. M. Farley, G. V. Fedotovich, F. E. Gray, D. Grigoriev, M. Grosse-Perdekamp, A. Grossmann, M. F. Hare, D. W. Hertzog, X. Huang, V. W. Hughes, M. Iwasaki, K. Jungmann, D. Kawall, B. I. Khazin, F. Krienen, I. Kronkvist, A. Lam, R. Larsen, Y. Y. Lee, I. Logashenko, R. McNabb, W. Meng, J. P. Miller, W. M. Morse, D. Nikas, C. J. G. Onderwater, Y. Orlov, C. S. Özben, J. M. Paley, Q. Peng, C. C. Polly, J. Pretz, R. Prigl, G. zu Putlitz, T. Qian, S. I. Redin, O. Rind, B. L. Roberts, N. Ryskulov, Y. K. Semertzidis, P. Shagin, Y. M. Shatunov, E. P. Sichtermann, E. Solodov, M. Sossong, L. R. Sulak, A. Trofimov, P. von Walter, A. Yamamoto, Measurement of the negative muon anomalous magnetic moment to 0.7 ppm, Phys. Rev. Lett. 92 (2004) 161802. doi:10.1103/PhysRevLett.92.161802. URL https://link.aps.org/doi/10.1103/PhysRevLett.92.161802
- [4] D. Stratakis, M. E. Convery, C. Johnstone, J. Johnstone, J. P. Morgan, D. Still, J. D. Crnkovic, V. Tishchenko, W. M. Morse, M. J. Syphers, Accelerator performance analysis of the fermilab muon campus, Phys. Rev. Accel. Beams 20 (2017) 111003. doi:10.1103/PhysRevAccelBeams.20.111003. URL https://link.aps.org/doi/10.1103/PhysRevAccelBeams.20.111003
- [5] L. Carver, D. Stratakis, Realistic modeling of a particle-matter-interaction system for controlling the momentum spread of muon beams, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 962 (2020) 163704. doi:10.1016/j.nima.2020.163704.
- [6] A. Yamamoto, Y. Makida, K. Tanaka, F. Krienen, B. Roberts, H. Brown, G. Bunce, G. Danby, M. G-Perdekamp, H. Hseuh, L. Jia, Y. Lee, M. Mapes, W. Meng, W. Morse, C. Pai, R. Prigl, W. Sampson, J. Sandberg, A. Steinmetz, The superconducting inflector for the bnl g-2 experiment, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 491 (2002) 23-40. doi:10.1016/S0168-9002(02)01232-9.
- [7] J. Bradley, J. Crnkovic, D. Stratakis, M. Syphers, Initial Studies into Longitudinal Ionization Cooling for the Muon g-2 Experiment (2018) TUPMK015doi:10.18429/JACoW-IPAC2018-TUPMK015.
- [8] D. Stratakis, R. B. Palmer, Rectilinear six-dimensional ionization cooling channel for a muon collider: A theoretical and numerical study, Phys. Rev. ST Accel. Beams 18 (2015) 031003. doi:10.1103/PhysRevSTAB.18.031003. URL https://link.aps.org/doi/10.1103/PhysRevSTAB.18.031003